Math 403/503 Spring 2024

Homework 4, due February 14

1. Let G be the additive group of real numbers. Let the action of $\theta \in G$ on the real plane $\mathbb{R}^2$ be given by rotating the plane counterclockwise about the origin through θ radians. Let P be a point on the plane other than the origin.

   (a) Show that this rule defines an action.

   (b) Describe geometrically the orbit containing P.

   (c) Find the stablizer subgroup G_P of P.

2. Let G be a group acting on a set X. Show that $gG_xg^{-1} = G_{g.x}$ for all $g \in G$ and all $x \in X$.

3. Let $G = A_4$ and let G act on itself by conjugation.

   (a) Determine the orbits (conjugacy classes) of each element in G.

   (b) For each $\sigma \in G$, find the stabilizer subgroup G_σ (centralizer).

4. Let $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ be the quaternion group of order 8.

   (a) Find $|Z(G)|$.

   (b) Find a representative g_i for each non-trivial conjugacy class of Q_8.

   (c) Compute $[G : C_G(g_i)]$ and verify that the class equation holds for G.

5. Suppose that the vertices of a regular hexagon are to be colored either cardinal or gold. How many ways can this be done up to a symmetry of the hexagon (from the dihedral group D_6)?